Keywords: mathematical model, overall efficiency, worm gear

Abstract. Overall efficiency represents an important qualitative property for devices used in manufacturing technology. This paper presents the mathematical modelling of calculating worm gear overall efficiency. The studied worm gear types goes into numerous manufacturing devices component. Mathematical model validation was experimentally performed on a tool sharpening device with a helicoidally profile. The analytical calculated results were closed to experimentally determined results, which partially validates the model proposed in the paper.

Introduction

Overall efficiency ($\eta$) is the ratio between effective work and mechanical work done by device. When using any machine or device, a part of the consumed energy is lost such that the overall efficiency cannot be unitary, only sub unitary [1, 2].

The energy required for such operations is considered as an input of the process, which is partially transformed into useful work, and wastes while the rest is transformed into waste/lost heat. Machining utilize only a fraction of the consumed energy for the actual value-adding process, while the majority of the energy is used for creating stable process conditions and peripheral functions [3, 4, 5].

There are a large number of different types of helicoidal kinematic couplings that goes into a large number of machines and manufacturing devices that are used directly or indirectly in the production manufacturing such as: industrial robots, tools machines, devices and equipment, etc [6, 7, 8].

On this paper, in the analytical overall efficiency calculation, we took into account only the phenomena that occur on worm gear and in the experimental part the overall efficiency was quantitatively determined, taking into account all factors acting on the device.

The analytical model of overall efficiency

The device for which we will calculate the overall efficiency is schematically represented in figure 1 a) and physical device are shown in figure 1 b). For overall efficiency calculation, the mathematical modelling only uses worm gear driver parameters, ignoring other factors that are characteristically to parts that are part of the device (part qualitative aspects like dimensional precision and surface condition).

For worm gear efficiency calculation, we started from equation (1) that defines efficiency as ratio between effective work $L_u$ and mechanical work done $L_c$.

$$
\eta = \frac{L_u}{L_c}
$$

(1)
Fig. 1. Tool sharpening device, a) schematic variant b) realized variant

To calculate mechanical work done that is made using relation (2) is necessary to know the components: $F_T$ represent transverse force and $F_N$ axial force.

$$L_w = M_2 \cdot \varphi_2 = F_T \cdot \frac{m_2 \cdot z_2}{2} \cdot \eta_{arb.pr} \cdot \pi \cdot m_1 \cdot z_1 \cdot \tan \gamma \cdot \frac{1}{z_2}$$  \hspace{1cm} (2)

where, $\eta_{arb.pr}$ represents the main shaft bearing overall efficiency, $m_2$ represents the worm wheel module (10), $z_2$ worm wheel teeth number (4 in figure 1), $m_1$ gear module (1 in figure 1), $z_1$ gear teeth number (1-figure 1), $\gamma$ slide inclination angle (3 in figure 1).

To calculate the force $F_T$ is used relation (3) and for normal force, relation (4).

$$F_T = (F_{cr} - \mu \cdot F_N \cdot \cos \gamma) \cdot \tan \gamma$$  \hspace{1cm} (3)

$$F_N = \frac{F_{cr}}{\sin \gamma}$$  \hspace{1cm} (4)

The mechanical work done is calculated with relation (5) where $\mu$ represents friction coefficient from worm gear and $\varphi = 2 \cdot \pi$ represents the angle corresponding to a hand wheel complete revolution (2 in figure 1).

$$L_c = M_1 \cdot \varphi = F_{cr_1} \cdot \frac{m_1 \cdot z_1}{2} \cdot 2 \cdot \pi = \left( \frac{1 - \frac{\mu}{\sin \gamma}}{\frac{m_1 \cdot z_1}{2} \cdot 2 \cdot \pi} \right)$$  \hspace{1cm} (5)

$F_{cr}$ force is calculated using equation (6) where $M_1$ represents the developed hand wheel moment (2 in figure 1).

$$F_{cr} = \frac{2 \cdot M_1}{m_1 \cdot z_1}$$  \hspace{1cm} (6)

Substituting relations (2) and (5) in equation (1), overall efficiency calculation expression becomes equation (7).
\[ \eta = \frac{L_u}{L_c} = \frac{F_r \cdot \frac{m_2 \cdot z_2 \cdot \pi \cdot m_1 \cdot z_1 \cdot \tan \gamma \cdot 1}{z_2^2}}{F_r \cdot \frac{m_1 \cdot z_1 \cdot 2 \cdot \pi}{2}} = \frac{(F_{cr} - \mu \cdot F_N \cdot \cos \gamma) \cdot \tan \gamma \cdot m_2}{F_{cr} \cdot 2} = \frac{F_{cr} \cdot \left(1 - \mu \cdot \frac{1}{\sin \gamma} \cdot \cos \gamma\right) \cdot \tan \gamma \cdot m_2}{F_{cr} \cdot 2} \]

where \( \gamma \) is the slide inclination angle (3 in figure 1).

Using equation (7) we can determine the mechanical overall efficiency to worm gear with tilt and slide patina, which are part of the studied device (figure 1).

Using constructive variant characteristic parameters \((m_1 = 2, \ z_1 = 30, \ \gamma = 45^\circ, \ \mu = 0.08, \ m_2 = 1.5, \ z_2 = 45)\) shown in figure 2, we calculated, based on relation (7), the overall efficiency that resulted as being \(\eta = 0.59\).

Experimental part

Overall efficiency determination involves the evaluation between mechanical work done for mechanism actuation and mechanical work resulted at the mechanism output.

For forces determination it was used a piezoresistive transducer. The forces were measured at the mechanism input and output by direct measuring of each force and a difference measurement between input force and output force by using a differential method.

Objective

The main objective of our experimental work was to study the overall efficiency of using a tools sharpening device. The experimental research is necessary for overall efficiency calculation model validation, proposed in this paper.

Materials

For experimental research was used a tools sharpening device for which the overall efficiency was determined. The device is presented in figure 1 b) and has been designed and developed by the first author [9].

For forces determination was used a CP-151 type piezoresistive transducers, manufactured by IEE S.A. ZAE WEIErgewan, L-5326 Contern, Luxembourg. This transducer type was developed for automation applications, especially in automotive, but can have various applications in other areas of forces measurement. In this experiment the transducer it was used for determine indirect the forces applied to hand wheel, by measuring the electrical resistance of transducer.

For transducer resistance variation measurement, depending on applied force, an DT9205A digital ohmmeter was used on 0...20 Kohm range.

Experimental measurements

The plan of experimental measuring is presented in figure 2. On hand wheel, used for device training to generate helicoidal surfaces, it was welded a support to mount the input force measurement transducer (figure 3.a) and to test weight application, and also cinematic chain stiffness and efficiency determination.
A second transducer was installed at the device exit using a driving piece from device construction and a rectangular support for transducer mounting, caught with a M8 screw on the device table channel (figure 3.b).

In order to determine device input and output forces during device manoeuvring, it was necessary a measurement device calibration such that reading the voltage and resistance indicated by device to be able to determine the used force.

For calibration were used test weight with which the device was loaded with different forces (figure 3.a). For T1 transducer calibration, the following correspondences were established between acting force $F_1$ and resistance $R_1$ (it was measure the voltage and it was converting in resistance), that are represented in table 1. The same was done with T2 transducer and measurement results are presented in table 2.

<table>
<thead>
<tr>
<th>Force $F_1$ (N)</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance $R_1$ (kΩ)</td>
<td>11,03</td>
<td>9,75</td>
<td>8,02</td>
<td>7,45</td>
<td>6,52</td>
<td>5,21</td>
<td>3,94</td>
<td>2,8</td>
<td>2,3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Force $F_2$ (N)</th>
<th>5</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance $R_2$ (kΩ)</td>
<td>15,25</td>
<td>11,25</td>
<td>9,95</td>
<td>8,40</td>
<td>7,51</td>
<td>6,50</td>
<td>5,01</td>
<td>3,55</td>
<td>2,52</td>
</tr>
</tbody>
</table>

Based on these values, using GRAPH v4.4.2 software, by interpolation, the figure 4.a) curve was plotted for T1 transducer and 4.b) curve for T2 transducer.

<table>
<thead>
<tr>
<th>Applied force $F_1$ (N)</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured voltage (mV)</td>
<td>21,5</td>
<td>23,5</td>
<td>41,4</td>
<td>19,5</td>
<td>0,1</td>
<td>-20,5</td>
<td>-31,2</td>
<td>-23</td>
<td>-20,5</td>
</tr>
<tr>
<td>Resistance difference $R_2 - R_1$ (Ω)</td>
<td>215</td>
<td>235</td>
<td>414</td>
<td>195</td>
<td>1</td>
<td>205</td>
<td>312</td>
<td>230</td>
<td>205</td>
</tr>
</tbody>
</table>
After above calibration, measurements were made to determine forces using the two transducers, to assess the device efficiency.

**Results and discussion**

To determine the forces, we used a piezoresistive transducer with the maximum measurement range up to 400N. The forces were measured at the entry and exit of the device by direct measurement of each force, and a difference measurement between input force and output force using a differential method.

When using direct method to determine overall efficiency, after performing experimental tests on tool sharpening machine, the following results were obtained from sensors, presented in table 4.

<table>
<thead>
<tr>
<th>Applied force $F_1$ (N)</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured resistance $R_2$ (kΩ)</td>
<td>14,5</td>
<td>12,3</td>
<td>10,6</td>
<td>9,7</td>
<td>9,05</td>
<td>8,7</td>
<td>8,4</td>
<td>8,1</td>
<td>7,9</td>
</tr>
<tr>
<td>$F_2$ force (N)</td>
<td>4</td>
<td>10,8</td>
<td>23</td>
<td>30,7</td>
<td>38</td>
<td>41,2</td>
<td>46,5</td>
<td>52,5</td>
<td>56</td>
</tr>
<tr>
<td>Overall efficiency $\eta$</td>
<td>0,4</td>
<td>0,43</td>
<td>0,46</td>
<td>0,41</td>
<td>0,38</td>
<td>0,33</td>
<td>0,31</td>
<td>0,30</td>
<td>0,28</td>
</tr>
</tbody>
</table>

When using the differential method to determine the overall efficiency, after performing experimental tests on tool sharpening devices we obtained from sensors the following results presented in table 5.

<table>
<thead>
<tr>
<th>$F_1$ (N)</th>
<th>10</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_2-U_1$ (V)</td>
<td>1,743</td>
<td>1,123</td>
<td>0,961</td>
<td>0,842</td>
<td>0,801</td>
<td>0,788</td>
<td>0,769</td>
<td>0,757</td>
<td>0,702</td>
</tr>
<tr>
<td>$R_2$ (kΩ)</td>
<td>17,2</td>
<td>11,0</td>
<td>9,2</td>
<td>8,4</td>
<td>8,1</td>
<td>8,0</td>
<td>7,9</td>
<td>7,7</td>
<td>7,2</td>
</tr>
<tr>
<td>$F_2$ (N)</td>
<td>3,7</td>
<td>11,3</td>
<td>22,1</td>
<td>30,1</td>
<td>39,3</td>
<td>43,5</td>
<td>48,2</td>
<td>56,2</td>
<td>59</td>
</tr>
<tr>
<td>$\eta=F_2/F_1$</td>
<td>0,37</td>
<td>0,45</td>
<td>0,44</td>
<td>0,40</td>
<td>0,39</td>
<td>0,34</td>
<td>0,28</td>
<td>0,32</td>
<td>0,29</td>
</tr>
</tbody>
</table>

Based on overall efficiency results obtained in table 4 and 5 we could draw overall efficiency variation curves in the case of studied device. Figure 6 presents overall efficiency variation curves.
Figure 6 presents the two overall efficiency curves, to make a comparative analysis of the two measurement methods. The values are close enough (7% maximum difference), so the two methods are validated. Overall efficiency values range between 0.28 and 0.46 for both methods, with lower values when applied forces increase.

**Conclusion**

This paper presents the mathematical modelling of overall efficiency calculation method on the tool sharpening device. In overall efficiency analytical calculation we considered only factors that can be established with certainty. The results obtained by calculation based on mathematical model (the overall efficiency was $\eta = 0.59$) were compared with the experimental tests results (presented in table 4 where $\eta = 0.46 \div 0.28$ and table 5 where $\eta = 0.52 \div 0.28$). After comparing the results, we obtained an overall efficiency difference of 0.14-0.3, i.e. 23%-50% between analytical and experimental method. The difference is due to device manufacturing tolerance, measurement errors, friction phenomenon at the device contact surfaces, etc. From results we can conclude that overall efficiency analytical calculation is significantly influenced by machine or device components quality. Therefore the calculated overall efficiency in design stage of a device or machine is only an enlarge value, the actual overall efficiency being always smaller than the calculated one.

**References**


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